**Unit 3 Assignment 3**

Group 1- Cayley graph, Orbits

* Draw the Cayley graph of S\_3 and compare with that of D\_3 the group of symmetries of equilateral triangle.
* Compare the Cayley table of both S\_3 and D\_3.
* Write out the orbits of elements of the finite groups that we have seen so far.
* Write out the orbits of a few elements in Z, Q, R, which are infinite groups.
* Write the orbit of a few matrices in some of the matrix groups.

Group 2- Cayley graph, cycle graph

* Consider the permutation group with 4 elements. S\_4 called the symmetric group. Write out the Cayley table.
* In order to draw the Cayley graph, identify the generators. (There can be more than one choice of generators). Write the orbits of all the elements of this group. Use this to draw the Cayley graph
* Draw the cycle graph of S\_4.

Group 3 Subgroups

Subgroup is a subset of a group that also satisfies the axioms of a group.

* Look at the Cayley graphs of small groups encountered till now and identify some of its subgroups.   
  So far we have looked at cyclic groups C\_n, permutation groups S\_n, group of symmetries of n sided polygon D\_n.
* Examine infinite groups such as number systems, matrix groups and identify some subgroups.

Group 4: Generators, abelian group

* Consider a rectangle ABCD. The horizontal flip (H) exchanges the left two vertices A and D, also exchanges the right two vertices B and C. Now locate the centre of the rectangle, where the diagonals intersect. Rotation (R ) about 180 degrees interchange A and C the ends of a diagonal and similarly B and D. This is also a symmetry. Do H and R commute? Show that all symmetries can be written using H, R and their inverses. Do you get the same group as the one we got last class?
* To show that a group is abelian, it is enough to check the condition a\*b=b\*a, for all of its generators a, b.
* List out generators and the relations between them in following kinds of groups; Cyclic groups (Z\_n), permutation group S\_3, S\_4.

Group 5 Dihedral group

* Take the regular polygon. Name the vertices. Just like we did for the equilateral triangle, identify the symmetries. Write down all the symmetries. Identify the generators and the write the Cayley table for this and draw the Cayley diagram.
* Find the subgroups of the above group. Write the Cayley table for each of the subgroups. Does it resemble any of the groups encountered earlier? For example, does the group D\_3 with 6 elements ‘resemble’ any of the groups in this list? Can you give it a geometric interpretation?